The Co-operative University of Kenya

END OF SEMESTER EXAMINATIONS MAY-2019

EXAMINATION FOR THE DEGREE OF BACHELOR OF STATISTICS & INFORMATION TECHNOLOGY/ INFORMATION TECHNOLOGY

UNIT CODE: BMAT 1204

UNIT TITLE: DISCRETE MATHEMATICS

DATE: 3RD MAY, 2019 **TIME: 2:00 PM – 4:00 PM**

INSTRUCTIONS:

- Answer question **ONE (compulsory)** and any other **TWO** questions **Question one**

QUSTION ONE

(a) i. List the members of the set (1 mark)

$$A\{x|x \text{ is an integer such that } x^2 = 2\}$$

ii. Use set builder notation to give a description of the set (2 marks)

$$B = \{0,3,6,9,12\}$$

(b) Find the sets A and B if $A - B = \{1,5,7,8\}, B - A = \{2,10\}$ and $A \cap B = \{3,6,9\}$

(2 marks)

(c) If A= {1,2,3,4} find P(A), the power set of A (4 marks)

(d) Explain why is it true that any argument with a conclusion with the form $P \vee \neg p$ is valid? (4 marks)

(e) Let p(x) be the statement"$x + 1 > 2x$". If the domain consists of the integers, what are these truth values? (4 marks)

i. P(0)

ii. $\forall x P(x)$

iii. $\forall x \sim P(x)$

iv. $\exists x \sim P(x)$

(f) Let $f: \mathbb{Z} \to \mathbb{Z}$ with f(x)=2x+1 Determine whether f is one-to –one and onto (3 marks)

(g) Determine whether these conditional/conditional statements are true or false (4 marks)

i. If 1+1=2, then 2+2=5

ii. If 1+1=3, then dogs can fly can fly

iii. 0>1 if only if 1=2

(h) Show using mathematical induction that $2(4^n) + 1$ is divisible by 3 for $n \in \mathbb{N}$

(4 marks)

(i) Prove that if n is an integer and 3n+2 is even, then n is even using a proof by contraposition (3 marks)

QUESTION TWO

(a) Prove using mathematical induction that

$$1.2 + 2.3 + 3.4 + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3}$$ (6 marks)

(b) Prove by contradiction for all integers n if n^2 is odd then n is odd (3 marks)

(c) If $f, g: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 2x^2 + 1$ and $g(x) = 7x^2 + 6$ compute

i.$f \circ g$ (3 marks)

ii. g$\circ f$ (3 marks)

(d) Given that $f(x) = \frac{2x+5}{3x+7}$ then find f^{-1} (3 marks)

(e) Name the following laws of arithmetic

i. .$x(y+z) = xy + yz$ (1 mark)

ii. $(x+y)+z = x+(y+z)$ (1mark)

QUESTION THREE

(a) Test the validity of the argument below:

If $3-5 \geq 7$ then either it is rainy or Arsenal will win the league. It is not rainy or Arsenal will not win the league. $3-5 \ngeq 7$If and only if it is rainy. Therefore if Arsenal will win the league,$3-5 \geq 7$ (6 marks)

(b) Write the inverse , converse and contrapositive of the following statement " if $6 + 8 \geq 8$, then goats can dance" (3 marks)

(c) In a survey of 500 people, 285 are interested in football game, 195 are interested in hockey game, 115 are interested in basketball game, 45 in football and basketball, 70 in football and hockey and 50 in hockey and basketball games and 50 are not interested in any of these games. By use a Venn diagram determine :

i. How many people are interested in all the three games? (2 marks)

ii. How many are interested in exactly one of the three games? (2 marks)

iii. How many are interested in at most two of these games (2 marks)

(d) By use of a truth table, compute $((\sim p \rightarrow \sim r) \leftrightarrow (r \rightarrow p))$Hence conclude whether it is a tautology or not (3 marks)

(e) Find a counter example of these universally quantified where the domain of discourse is the set of real numbers

i. $\forall x(x^2 \neq x)$ (1 mark)

ii. $\forall x(x^2 \neq 2)$ (1 mark)

QUESTION FOUR

(a) If $A \subset B$ and $B \subset D$, show that $A \subset D$ (3 marks)

(b) Show that $A - B = A \cap B^c$ and hence simplify $(A - B) \cap (B - A)$ (6 marks)

(c) Show that the function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = |x|$ is not one to one.

(d) Determine the domain of each of the following functions

i. $f(x) = \sqrt{16 - x^2}$ ` (2 marks)

ii. $g(x) = \frac{1}{(x-2)(x_3)}$ (2 marks)

(e) Determine the power set of $A = \{1, \{1,2\}\}$ (2 marks)

(f) If $B = \{\emptyset, \{a\}, 2\}$ and $D = \{a, b\}$, determine $A \times B$ (3 marks)

QUESTION FIVE

(a) How many different license plates can be made if each plate contains a sequence of three uppercase English letters followed by three digits (and no sequences of letters are prohibited, even if they are obscene)? (3 marks)

(b) In a discrete mathematics class every student is a major in computer science or mathematics, or both. The number of students having computer science as a major (possibly along with mathematics) is 25; the number of students having mathematics as a major (possibly along with computer science) is 13; and the number of students majoring in both computer science and mathematics is 8. How many students are in this class? (4 marks)

(c) State and prove generalized pigeon hole principle (2 marks)

(d) A total of 1232 students have taken a course in Spanish, 879 have taken a course in French, and 114 have taken a course in Russian. Further, 103 have taken courses in both Spanish and French, 23 have taken courses in both Spanish and Russian, and 14 have taken courses in both French and Russian. If 2092 students have taken at least one of Spanish, French, and Russian. Determine

i. How many students have taken a course in all three languages? (3 marks)

ii. How many students have taken Spanish, French and Russian only (3 marks)

iii. How many students have taken at least two languages (3 marks)

(e) How many students have taken at most two languages (2 marks)