

**THE CO-OPERATIVE UNIVERSITY  
OF  
KENYA**

**UNIVERSITY EXAMINATIONS 2019/2020**

**EXAMINATION FOR THE DEGREE OF BACHELOR SCIENCE COMPUTER SCIENCE  
AND BACHELOR SCIENCE INFORMATION TECHNOLOGY SECOND YEAR FIRST  
SEMESTER**

**BMAT 1205: CALCULUS 1**

DATE: DECEMBER 2019

TIME: 2 HOURS

***INSTRUCTIONS: Attempt Question One and any other Two Questions.***

**QUESTION ONE (30 MARKS)**

(a) Differentiate the following functions. [8 marks]

(i)  $y = (x^2 + 1)(x^3 + 3)$

(ii)  $y = x^4 - 2x^2 + 2$

(iii)  $y = \sin x \cos x$

(iv)  $y = \frac{\ln x}{x^3}$

(b) Define a function hence given the functions

$f(x) = \frac{5-2x}{1+3x}$  and  $g(x) = \frac{x}{1+5x}$ . Find fog and state its domain. [5 marks]

(c) Evaluate the following limits;

(i) Proof

$\lim_{\theta \rightarrow 0} \left( \frac{\sin \theta}{\theta} \right) = 1.$  [3 marks]

(ii)  $\lim_{x \rightarrow \infty} \left( \frac{6-2x^3-x^4}{3x^3+2x-4} \right).$  [3 marks]

(d) A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1ft per second, How fast is the top of the ladder sliding down when the bottom of the ladder is 6 ft from the wall. [5 marks]

(e) Find the first derivative of the function given by  $y = \ln \left( \frac{x}{(x^3+1)^{\frac{1}{3}}} \right),$  [6 marks]

**QUESTION TWO (20 MARKS)**

- (a) Find  $\frac{d^3y}{dx^3}$  of; [4 marks]  
 $y = 6x^2 - 10x - 5x^2$   
 $y = \frac{x^2}{1+x}$
- (b) If  $y = \frac{\cos x}{x}$ , prove that  $\frac{d^2y}{dx^2} + \frac{2}{x} \frac{dy}{dx} + y = 0$ . [3 marks]
- (c) Find the largest possible area of an isosceles triangle if the length of its two equal sides is 10. [6 marks]
- (d) Air is being pumped into a spherical balloon so that its volume increases at a rate of  $100\text{cm}^3/\text{sec}$ . How fast is the radius of the balloon increasing when the diameter is 50cm. [7 mark]

### QUESTION THREE (20 MARKS)

- (a) Evaluate the following limit using factorization method;  
 $\lim_{x \rightarrow 3} \frac{6x^3 - 17x^2 - 4x + 3}{x - 3}$ . [6 marks]
- (b) Evaluate the integral:  
 $\int 3x^3 \exp^{x^3-1} dx$ . [6 marks]
- (c) An oil tanker has an accident and oil pours out at a rate of 150 gallons per minute. Suppose that the oil spreads onto a river in a circle at a thickness of  $\frac{1}{120}$ . Given that 1 ft is equal to 75 gallons, determine the rate at which the radius of the spill is increasing when the radius reach 500ft. [8 marks]

### QUESTION FOUR (20 MARKS)

- (a) A particle moves on an horizontal line according to  $S(t) = t^4 - 6t^3 + 12t^2 - 10t + 3$
- (i) Find when is the speed increasing or decreasing. [3 marks].
  - (ii) When the particle is stationary. [2 marks]
  - (iii) Find the total distance travelled in the first 3s of the motion. [2 marks]
- (b) If  $y = \sqrt[3]{x}$ , estimate the value of  $\sqrt[3]{1001}$ . [4 marks]
- (c) A public transportation company has been experiencing on possibility of developing a system of charging fares. The demand functions, which expresses the ridership as a function of fare charged is given below:  
 $Q = 10000 - 125p$   
 Where Q equals the average number of riders per hour and p equals the fare in shillings
- (i) Determine the fare, which should be charged in order to maximize hourly bus fare revenue [4 marks]
  - (ii) Whats the expected maximum revenue? [3 marks]
  - (iii) How many riders per hour are expected under this figure? [2 marks]