The Co-operative University of Kenya

END OF SEMESTER EXAMINATIONS

EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN INFORMATION TECHNOLOGY, BACHELOR OF SCIENCE IN STATISTICS IN INFORMATION SCIENCE AND BACHELOR OF SCIENCE IN COMPUTER SCIENCE

COURSE CODE: BMAT 1101

UNIT TITLE: BASIC MATHEMATICS

DATE: DECEMBER, 2019 Time: 2 Hours

INSTRUCTIONS

1. Answer question **ONE** and any other **TWO (2)** questions
2. Scientific Calculators and non-programmable calculators may be used
3. Use the provided statistical tables where applicable

QUESTION ONE

1. (a) Express $-1+i$ in polar coordinates form. (**5** *marks*)

   (b) Express as partial fraction $\dfrac{2x+3}{(x-2)(x^2+x+1)}$. (**5** *marks*)

   (c) If $A=\big(1,3,5\big)$, $B=\big(3,4\big)$, find $A\times B$,$B\times B$

   (d) Let A and B be subsets of a universal set U and suppose $n(U)=100$ $n(A)=60$ $n(B)=40$ and $n(A\cap B)=20$. Compute using the Venn diagram

   i. $n(A\cup B)$ (**2** *marks*)

   ii. $n(A\cap B^c)$ (**1** *marks*)

   iii. $A^c\cap B$ (**1** *marks*)

   (e) Solve for x in the equation $\log_4 x+\log_2 x=6$. (**4** *marks*)

   (f) The expression x^3+ax+b leaves a remainder of -44 when divided by $x+3$, a remainder of 6 when divided by $(x-2)$. Calculate the value of a and b. (**6** *marks*)

   (g) The third and fifth term of a geometric series are 144 and 324.Find the seventh term. (**5** *marks*)

QUESTION TWO

2. (a) The function g is defined by $g(x)=\dfrac{x+1}{x-2}$. Find and simplify an expression for

   i. g^2

   ii. g^{-1}

   The function h is defined by $h(x)=\dfrac{ax+3}{x}$.
   Given that $hg^{-1}(4)=6$, calculate the value of a. (**10** *marks*)

(b) If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$, prove that
$x + y + z = xyz$. (**5** *marks*)

(c) Given that $(x+6), x, (x-3)$ are the first three terms of a geometric progression. Calculate the value of

i. x (**3** *marks*)

ii. the fifth term. (**2** *marks*)

QUESTION Three

3. (a) Given that $x^3 + 4x^2 - ax + b$ is exactly divisible by $(x+2)$ but leaves a remainder a^3 when divided by $(x-a)$, calculate the values of a and b. (**6** *marks*)

(b) Find the term independent of x in the expansion of $\left(\frac{4x^2}{3} - \frac{3}{2x}\right)^9$. (**4** *marks*)

(c) Find the general solution of the following trigonometry equation
$\sqrt{3}\cos x + \sin x = \sqrt{2}$. (**5** *marks*)

(d) If $A = (x : x^2 - 6x + 5 =)$ $\quad B = (1, 2) \quad C = (2, 5)$, Find $(A - B) \times (B - C)$.(**5** *marks*)

QUESTION Four

4. (a) In a college, 60 members of the staff drink tea and 50 members of the staff drink coffee. If the total staff of the college is 85, how many members of the staff drink both tea and coffee. (Assume every staff member drinks tea or coffee). (**4** *marks*)

(b) If α and β are roots of the equation $3x^2 - 2x + 1 = 0$. Find

i. $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$ (**5** *marks*)

ii. $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$ (**4** *marks*)

(c) Solve for x in the equation $tan^{-1}x + tan^{-1}\frac{1}{3} = \frac{\pi}{4}$. (**3** *marks*)

(d) Solve the simultaneous equations

$$y + 4 = x$$
$$y^2 + 17 = 2x^2$$

(**4** *marks*)

QUESTION FIVE

5. (a) Find the equation of the line which is parallel to $3y = 4x + 5$ and which passes through the mid-point of the line joining (-2,3) and (5,9). (**4** *marks*)

(b) The expression $ax^3 - 4x^2 + bx - 5$ has a factor of (x+1) and leaves a remainder of 4 when divided by $(x-3)$. Calculate the remainder when the expression is divided by $(x-2)$. (**6** *marks*)

(c) The sum of n terms of an arithmetic progression is given by the formula $S_n = 2n^2 + n$. Find

i. the first term (**1** *mark*)

ii. the common difference (**4** *marks*)

iii. the tenth term. (**1** *mark*)

(d) What sum (P) should be invested in a loan association at 4 compounded semiannually, so that the compound amount(S) will be 500 dollars at the end of 3.5 years?

Solutions to Exam

1a.

$$-1 + i = rcos\theta + rsin\theta$$

$$-1 = rcos\theta$$
$$1 = rsin\theta$$
$$\sqrt{2} = r$$
$$cos\theta = \frac{-1}{\sqrt{2}}$$
$$\theta = \frac{3\pi}{4}$$
$$-1 + i = \sqrt{2}(cos\frac{3\pi}{4} - i\sin\frac{3\pi}{4})$$

1b.

$$\frac{2x+3}{(x-2)(x^2+x+1)} = \frac{A}{x-2} + \frac{Bx+c}{x^2+x+1}$$

$$2x + 3 = A(x^2 + x + 1) + (Bx + c)(x - 2)$$

$$A + B = 0$$
$$A - 2B + c = 2$$
$$A - 2c = 3$$
$$A = 1, B = -1, C = -1$$

1c. $A \times B = (1,3),(1,4),(3,4),(3,3),(5,3),(5,4)$ $,B \times B = (3,3),(3,4),(4,4),(4,3)$

1(d)i. $n(A \cup B) = 60 + 40 - 20 = 80$ for correct Venn diagram

1(d)ii. By use of Venn diagram $n(A \cap B^c) = 40$

1(d)iii. from the Venn diagram $n(A^c \cap B) = 20$

1e. $\frac{logx}{log4} + \frac{logx}{log2} = 6, \frac{logx + 2logx}{2log2} = 6, 3logx = 12log2, x = 16$

1f. $f(-3) = -44$ and $f(2) = 6$ which gives $-3a + b = -17$ and $2a + b = -2$ $, a = 3, b = -8$

1g. $ar^2 = 144$ $,ar^4 = 324$ $,r^2 = \frac{9}{4}$ $, a = \frac{144}{2.25} = 64$ $, ar^6 = 729$

2(a)i. $g^2 = \frac{\frac{x+1}{x-2} + 1}{\frac{x+1}{x-2} - 2} = \frac{2x-1}{-x+5}$

2(a)ii. $y = \frac{x+1}{x-2}$ $x = \frac{2y+1}{y-1}$ $g^{-1}(x) = \frac{2x+1}{x-1}$

2a. $g^{-4} = 3$ $h(3) = \frac{3a+3}{3} = 6$ $a + 1 = 5$ $a = 5$

2b.

$$tan^{-1}x = A, tan^{-1}y = B, tan^{-1}z = C$$

therefore

$$x = tanA, y = tanB, z = tanC$$

$$A + B + C = \pi tan(A + B) = tan(\pi - c)$$

$\frac{tanA - tanB}{1 - tanAtanB} = \frac{0 - tanC}{1 - 0}$ $\frac{x+y}{1-xy} = -z$ $x + y + z = xyz$

2(c)i. $\frac{x}{x+6} = \frac{x-3}{x}$ $0 = 3x - 18$ $x = 6$

2(c)ii. $12(\frac{1}{2}^4) = \frac{3}{4}$

3a.

$$f(-2) = 0 = -8 + 4 + 2a + b$$

$$f(a) = a^3 = a^3 + 4a^2 - a^2 +$$

$2a + b = 4$
$3a^2 + b = 0$ $3a^2 - 2a - 4 = 0$ $a = \frac{-2}{3}$ or $a = 2$ $b = \frac{-4}{3}$ or $b = -12$

3b.

$$\binom{9}{r}(\frac{4x^2}{3})^{9-r}(\frac{-3}{2x})^r$$

$$18 - 2r - r = 0$$

$$r = 6$$

$$\binom{9}{6}(\frac{4}{3})^3(\frac{-3}{2})^6$$

$$-2268$$

3c. divide through by

$$\sqrt{(3+1)} = 2$$

$$\cos\alpha = \frac{\sqrt{3}}{2}\sin\alpha = \frac{1}{2}$$

$$\cos(x - \frac{\pi}{6}) = \frac{1}{\sqrt{2}}$$

$$x - \frac{\pi}{6} = 2n \pm \frac{\pi}{4}$$

$$x = (2n\pi \pm \frac{\pi}{4}) + \frac{\pi}{6}$$

3d. $(x-1)(x-5), A = (1,5)$,$(A-B) = (5)$,$(B-C) = 1$, $(A-B)\times(B-C) = (5,1)$

4a. $n(T) = 60, n(C) = 50$,$n(T\cap C) = x$,$n(T\cap C) = n(T) + n(C) - n(T\cup C)$,$110 - 85 = 25$

4(b)i. $\alpha + \beta = \frac{2}{3}, \alpha\beta = \frac{1}{3}, \frac{\beta^2+\alpha^2}{(\alpha\beta)^2} = \frac{(\beta+\alpha)^2 - 2\alpha\beta}{(\alpha\beta)^2} = \frac{\frac{4}{9} - \frac{2}{3}}{\frac{1}{9}} = -2$

4(b)ii. $\frac{\beta^3+\alpha^3}{\beta} = \frac{(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)}{}(\alpha\beta) = \frac{-10}{9}$

4c. $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B} = \frac{x + \frac{1}{3}}{1 - \frac{x}{3}} = 1, x = 1$

4d. $y^2 + 17 = 2(y^2 + 8y + 16)$, $y^2 + 16y + 15 = 0, y = -15, x = -11; y = -1, x = 3$

5a. $y = \frac{4}{3}x + \frac{5}{3}$,Mid-point$(\frac{3}{2}, 6)$,$\frac{y-6}{x - \frac{3}{2}} = \frac{4}{3}$,$3y - 4x = 12$

5b. $f(-1) = 0$, $a + b = -9$,$f(3) = 4$,$9a + b = 15$,$a = 3, b = -12$, $f(2) = -21$

5(c)i. $S_1 = 2 + 1 = 3$,$a = 3$

5(c)ii. $S_2 = 10$,$S_3 = 21$, $S_4 = 36$, the series is 3,7,11,15.... $d = 4$

5(c)iii. $3 + 9 \times 4 = 39$

5d. Since there two interest periods per year ,$n = 2(3.5) = 7$ and the interest per period is $i = \frac{1}{2}(0.04) = 0,02$,$S = P(1+i)^n$ or $P = S(1+i)^{-n} = P = 500(1+0.02)^{-7} = 435.28$