



## The Co-operative University of Kenya

**END OF SEMESTER EXAMINATIONS**

**EXAMINATION FOR DIPLOMAS IN YR 1 SEM 1**

**COURSE CODE: DUCU 1104**

**UNIT TITLE: FOUNDATIONS OF MATHEMATICS**

**DATE: December, 2019**

**Time: 2 Hours**

### INSTRUCTIONS

1. Attempt question **ONE** and any other **TWO (2)** questions
2. Scientific Calculators and non-programmable calculators may be used

1. (a) Define the following terms as used in sequence and series;
  - i. Sequence. (1 marks)
  - ii. Series. (1 marks)
  - iii. Arithmetic Progression. (2 marks)
  - iv. Geometric Progression. (2 marks)
- (b) Simplify;
 
$$\frac{6(y^4)^5}{12y^4y^5}$$
(3 marks)
- (c) Find the derivative of the following functions.
  - i.  $y = 15x^{100} - 3x^{12} + 5x - 46$  (3 marks)
  - ii.  $h(t) = \frac{2t^5 + t^2 - 5}{t^2}$  (4 marks)
- (d) In a survey of 60 people it was found that 25 read Newsweek magazine, 26 read Time, and 25 read Fortune. Also 9 read both Newsweek and Fortune, 11 read both Newsweek and Time, 8 read both Time and Fortune, and 8 read none of these three magazines.
  - i. Find the number of people who read all three magazines. (6 marks)
  - ii. Construct the corresponding Venn diagram. (6 marks)
  - iii. Determine the number of people who read exactly on magazine. (2 marks)
2. (a) In a certain Algebra class there is a total of 350 possible points. These points come from 5 homework sets that are worth 10 points each and 3 hour exams that are worth 100 points each. A student has received homework scores of 4, 8, 7, 7, and 9 and the first two exam scores are 78 and 83. Assuming that grades are assigned according to the standard scale and there are no weights assigned to any of the grades is it possible for the student to receive an A in the class and if so what is the minimum score on the third exam that will give an A? What about a B? Using the standard scale if the grade percentage is 0.9 or higher the student will get an A. Likewise, if the grade percentage is between 0.8 and 0.9 the student will get a B. (7 marks)

- (b) Jane gave money to a charity over a 20-year period, from year 1 to year 20 inclusive. She gave \$ 150 in year 1, \$ 160 in year 2, \$ 170 in year 3 and so on, so that the amount of money she gave each year formed an arithmetic sequence.
- Find the amount of money she gave in year 10. (3 marks)
  - Calculate the total amount of money she gave over the 20-year period. (4 marks)
  - Kelvin also gave money to the charity over the same 20-year period. He gave \$A in the year 1 and the amount of money he gave each year increased, forming an arithmetic sequence with common difference \$30. The total amount of money that Kelvin gave over 20-year period was twice the total amount of money that Jane gave.  
Calculate the value of A. (5 marks)
3. A class has 7 male and 5 female representatives. A committee is to be formed from the representatives. In how many ways can a 4-member committee be formed so that it is composed of; (20 marks)
- Any 4 representatives
  - 4 male representatives
  - 3 male and 1 female representative
  - 2 male and 2 female representatives
  - At least 2 male representatives.
4. (a) Given the matrices;  $C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$  and  $D = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$  Show that  $CD \neq DC$  (5 marks)
- (b) The prices of two commodities are represented by the following two equations. Find the values of  $x$  and  $y$  using **matrix inverse** method. (6 marks)
- $$2x - 2y - 3 = 0$$
- $$8y = 7x + 2$$
- (c) Write down the expansion of  $(x + \frac{1}{2}y)^3$  using the binomial theorem. (6 marks)
- (d) Simplify;  $\frac{10\sqrt{3}}{\sqrt{5}}$  (3 marks)
5. (a) The production cost in dollars, per week of producing  $x$  widgets is given by;
- $$C(x) = 4000 - 32x + 0.08x^2 + 0.00006x^3$$
- and the demand function for the widgets is given by,
- $$p(x) = 250 + 0.02x - 0.001x^2$$
- Determine the marginal cost, marginal revenue and marginal profit when  $x = 200$  and  $x = 400$ ? What does the numbers tell you about the cost, revenue and profit? (12 marks)
- (b) Evaluate the following integral. (4 marks)

$$\int 12t^7 - t^2 - t + 3 \, dt$$

(c) Evaluate the following integral.

(4 marks)

$$\int_{-2}^1 5z^2 - 7z + 3 \, dz$$

## Solutions to Exam

**1(a)i.** A sequence is a set of numbers written in a particular order.

**1(a)ii.** A series is a sum of the terms in a sequence.

**1(a)iii.** An arithmetic progression, or AP, is a sequence where each new term after the first is obtained by adding a constant  $d$ , called the common difference, to the preceding term.

**1(a)iv.** A geometric progression, or GP, is a sequence where each new term after the first is obtained by multiplying the preceding term by a constant  $r$ , called the common ratio.

**1b.**

$$\begin{aligned} &= \frac{6y^{20}}{12y9} \\ &= \frac{y^{20}}{2y9} \\ &= \frac{y^{11}}{2} \end{aligned}$$

**1(c)i.**

$$y' = 1500x^{99} - 36x^{11} + 5$$

(3 marks)

**1(c)ii.**

$$\begin{aligned} h(t) &= 2t^3 + 1 - 5t^{-2} \\ h'(t) &= 6t^2 + 10t^{-3} \end{aligned}$$

(4 marks)

**1(d)i.** We introduce the set  $U$  of all surveyed people and its subsets  $N$ ,  $T$ , and  $F$  of all people who read Newsweek, Time, and Fortune, respectively. The conditions of the problem then can be written as;

$$n(U) = 60$$

$$n(N) = 25, \quad n(T) = 26, \quad n(F) = 25,$$

$$n(N \cap F) = 9, \quad n(N \cap T) = 11, \quad n(F \cap T) = 8$$

$$n(N' \cap T' \cap F') = n[(N \cup T \cup F)'] = 8$$

$$n(N \cup T \cup F) = n(N) + n(T) + n(F) - n(N \cap T) - n(N \cap F) - n(T \cap F) + n(N \cap T \cap F)$$

$$n(N \cup T \cup F) = n(U) - n[(N \cup T \cup F)'] = 60 - 8 = 52$$

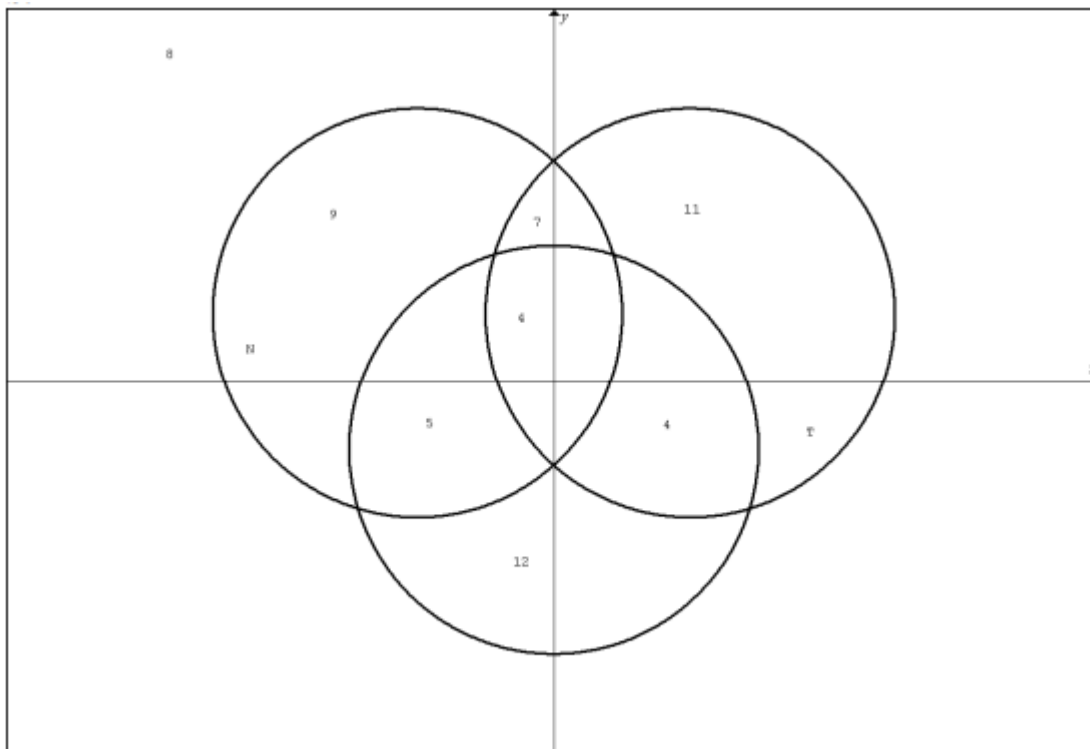
Plugging in the numbers in the formula, we have;

$$52 = 25 + 26 + 25 - 11 - 9 - 8 + n(N \cap T \cap F),$$

$$\text{hence } n(N \cap T \cap F) = 4$$

Four people read all the magazines.

1(d)ii.



✓ (6 marks)

Figure 1: Venn diagram

1(d)iii. From the Venn diagram we see that the number of people reading exactly one of these two magazines is  
 $9 + 11 + 12 = 32$ .

2a. Let  $p$  be the minimum required score on the third exam.

Since there are no weights on the grades, the grade will be computed using the formulae;

$$\text{grade percentage} = \frac{\text{actual points}}{\text{total points}} \quad (1)$$

And we know the total points is 350 and the student has a total points including the third exam of;

$$4 + 8 + 7 + 7 + 9 + 78 + 83 + p = 196 + p$$

The smallest possible percentage for an A is 0.9 and so if  $p$  is the minimum required score on the third exam for an A we will have the following equation.

$$0.9 = \frac{196 + p}{350}$$

This is a linear equation that we will need to solve for  $p$ .

$$196 + p = 0.9(350) = 315$$

$$\implies p = 315 - 196 = 119$$

So, the minimum required score on the third exam is 119. This is a problem since the exam is worth only 100 points. In other words, the student will not be getting an A in the Algebra class. Now let's check if the student will get a B. In this case the minimum percentage is 0.8. So, to find the minimum required score on the third exam for a B we will need to solve,

$$0.8 = \frac{196 + p}{350}$$

Solving for  $p$  gives;

$$\begin{aligned} 196 + p &= 0.8(350) = 280 \\ \implies p &= 280 - 196 = 84 \end{aligned}$$

So, it is possible for the student to get a B in the class. All that the student will need to do is get at least an 84 on the third exam.

**2(b)i.**

$$\begin{aligned} &150, 160, 170, \dots \\ &a = 150 \\ &d = 10 \\ &n^{\text{th}} \text{ term} = a + d(n - 1) \\ \text{Amount in 10 years} &= 150 + 10(10 - 1) \\ &= 150 + 90 \\ &= \$240 \end{aligned}$$

**2(b)ii.**

$$\begin{aligned} \text{Total amount } S_n &= \frac{n}{2}(2a + (n - 1)d) \\ &= \frac{20}{2}(2(150) + (20 - 1)10) \\ &= 10(300 + 190) \\ &= \$4900 \end{aligned}$$

**2(b)iii.**

$$\begin{aligned} &d = 30 \\ \text{Total given by Kelvin} &= \frac{20}{2}(2A + 19(30)) \\ &= 20A + 5700 \\ \text{Therefore } 20A + 5700 &= 2 * 4900 \\ 20A &= 4100 \\ A &= 205 \end{aligned}$$

**3a.**

$$\begin{aligned} &= \binom{7}{4} + \binom{5}{4} \\ &= 35 + 5 \\ &= 40 \text{ ways} \end{aligned}$$

3b.

$$= \binom{7}{4}$$

$$= 35 \text{ways}$$

3c.

$$= \binom{7}{3} * \binom{5}{1}$$

$$= 35 * 5$$

$$= 175 \text{ways}$$

3d.

$$= \binom{7}{2} * \binom{5}{2}$$

$$= 21 * 10$$

$$= 210 \text{ways}$$

3e. There are three possible ways;

2 males and 2 females or 3 males and 1 female or 4 males .

The combinations are given as;

$$= \binom{7}{2} * \binom{5}{2} + \binom{7}{3} * \binom{5}{1} + \binom{7}{4}$$

$$= (21 * 10) + (35 * 5) + 35$$

$$= 210 + 175 + 35$$

$$= 420 \text{ways}$$

4a.  $CD = \begin{bmatrix} 13 & 19 \\ 27 & 43 \end{bmatrix}$  but  $DC = \begin{bmatrix} 16 & 22 \\ 27 & 40 \end{bmatrix}$  so  $CD \neq DC$

4b. First write the equations in standard form then express them in matrix notation.

$$\begin{bmatrix} 2 & -2 \\ 7 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

$$\begin{vmatrix} 2 & -2 \\ 7 & 8 \end{vmatrix} = -2$$

$$-\frac{1}{2} \begin{bmatrix} -8 & 2 \\ -7 & 2 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 3.5 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -1 \\ 3.5 & -1 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \end{bmatrix} = \begin{bmatrix} 14 \\ -12.5 \end{bmatrix}$$

Therefore  $x = 14$  and  $y = 12.5$

4c.

$$\begin{aligned}
 \left(x + \frac{1}{2}y\right)^3 &= \sum_{k=0}^3 \binom{3}{k} x^{3-k} \left(\frac{1}{2}y\right)^k \\
 &= \binom{3}{0} x^3 + \binom{3}{1} x^2 \left(\frac{1}{2}y\right) + \binom{3}{2} x \left(\frac{1}{2}y\right)^2 + \binom{3}{3} \left(\frac{1}{2}y\right)^3 \\
 &= (1)x^3 + 3x^2 \left(\frac{1}{2}y\right) + 3x \left(\frac{1}{4}y^2\right) + \left(\frac{1}{8}y^3\right) \\
 &= x^3 + \frac{3}{2}x^2y + \frac{3}{4}xy^2 + \frac{1}{8}y^3
 \end{aligned}$$

4d.

$$\begin{aligned}
 &= \frac{10\sqrt{3}}{\sqrt{5}} * \frac{\sqrt{5}}{\sqrt{5}} \\
 &= \frac{10\sqrt{15}}{5} \\
 &= 2\sqrt{15}
 \end{aligned}$$

5a. The revenue and profit functions are given as;

$$\begin{aligned}
 R(x) &= xp(x) = 250x - 0.02x^2 - 0.001x^3 \\
 P(x) &= R(x) - C(x) \\
 &= -4000 + 282x - 0.06x^2 - 0.00106x^3
 \end{aligned}$$

The marginal functions are given as;

$$\begin{aligned}
 C'(x) &= -32 + 0.16x + 0.00018x^2 \\
 R'(x) &= 250 + 0.04x - 0.003x^2 \\
 P'(x) &= 282 - 0.12x - 0.00318x^2
 \end{aligned}$$

The marginal costs when 200 widgets are produced are;

$$C'(200) = 7.2, \quad R'(200) = 138, \quad P'(200) = 130.8$$

The marginal costs when 400 widgets are produced are;

$$C'(400) = 60.8, \quad R'(400) = -214, \quad P'(400) = -274.8$$

5b.

$$= \frac{3}{2}t^8 - \frac{1}{3}t^3 - \frac{1}{2}t^2 + 3t + c$$

(4 marks)

5c.

$$\begin{aligned}
 &= \frac{5}{3}z^3 - \frac{7}{2}z^2 + 3z \Big|_{-2}^1 \\
 &= \frac{7}{6} - \frac{-100}{3} \\
 &= \frac{69}{2}
 \end{aligned}$$