



The Co-operative University of Kenya

**SUPPLEMENTARY/SPECIAL EXAMINATIONS**

**EXAMINATION FOR DIPLOMAS IN YR 1 SEM 1**

**COURSE CODE: DUCU 1104**

**UNIT TITLE: FOUNDATIONS OF MATHEMATICS**

**DATE: December, 2019**

**Time: 2 Hours**

**INSTRUCTIONS**

1. Attempt question **ONE** and any other **TWO (2)** questions
2. Scientific Calculators and non-programmable calculators may be used

1. (a) Define the following terms as used in set theory and give the denotation. (6 marks)
  - i. Complement
  - ii. Intersection
  - iii. Union
- (b) A clock strikes once when its hour hand is at 1, twice when its at 2 and so on. How many times does the clock strikes in six hours? (7 marks)
- (c) Solve the following quadratic equations using factorization method.
  - i.
 
$$x^2 - x - 12$$

(3 marks)
  - ii.
 
$$x^2 + 40 = 14x$$

(3 marks)
- (d) Simplify;
 
$$(3y^2t)^2 \div (3yt^2)^{-2}$$

(4 marks)
- (e) Find the derivative of the following functions.
  - i.  $y = 15x^{100} - 3x^{12} + 5x - 46$  (3 marks)
  - ii.  $h(t) = \frac{2t^5 + t^2 - 5}{t^2}$  (4 marks)
2. (a) A company predicts a yearly profit of \$ 120000 in the year 2017. The company predicts that the yearly profit will rise by 5%. The predicted yearly profit forms a geometric sequence with a common ratio of 1.05.
  - i. Show that the predicted profit in the year 2020 is 138915 (2 marks)
  - ii. Find the first year in which the yearly predicted profit exceeds \$ 200000. (5 marks)

- iii. Find the total predicted profit for the year 2017 to 2027 inclusive giving your answer to the nearest dollar (3 marks)
- (b) A company, which is making 200 mobile phones each week, plans to increase its production. The number of mobile phones produced is to be increased by 20 each week from 200 in week 1 to 220 in week 2, 240 in week 3 and so on until it is producing 600 in week N.
- i. Find the value of N. (4 marks)
  - ii. The company then plans to continue to make 600 mobile phones each week. Find the total number of mobile phones that will be made in the first 52 weeks starting from and including week 1. (6 marks)
3. (a) From a group of 7 men and 6 women, five persons are to be selected to form a committee so that at least 3 men are there on the committee. In how many ways can it be done? (5 marks)
- (b) Compute the value of the following limit. (5 marks)

$$\lim_{x \rightarrow 2} \frac{x^2 + 4x - 12}{x^2 - 2x}$$

- (c) Given a system of linear equations, (10 marks)
- $$\begin{aligned} x_1 - 2x_2 + x_3 &= 3 \\ 2x_1 + x_2 - x_3 &= 5 \\ 3x_1 - x_2 + 2x_3 &= 12 \end{aligned}$$
- Use the inverse matrix to solve the system of equations.
4. (a) In a survey of 60 people it was found that 25 read Newsweek magazine, 26 read Time, and 25 read Fortune. Also 9 read both Newsweek and Fortune, 11 read both Newsweek and Time, 8 read both Time and Fortune, and 8 read none of these three magazines.
- i. Find the number of people who read all three magazines. (6 marks)
  - ii. Construct the corresponding Venn diagram. (6 marks)
  - iii. Determine the number of people who read exactly on magazine. (2 marks)
- (b) Simplify;  $(\sqrt{20} - \sqrt{5})(\sqrt{20} + \sqrt{5})$  (3 marks)
- (c) Simplify;

$$\left(\frac{5a}{b^2}\right)^2$$

(3 marks)

5. (a) The weekly cost to produce  $x$  widgets is given by;

$$C(x) = 75000 + 100x - 0.03x^2 + 0.000004x^3 \quad 0 \leq x \leq 10000$$

and the demand function for the widgets is given by,

$$p(x) = 200 - 0.005x \quad 0 \leq x \leq 10000$$

Determine the marginal cost, marginal revenue and marginal profit when 2500 widgets are sold and when 7500 widgets are sold. Assume that the company sells exactly what they produce. (12 marks)

- (b) In a certain Algebra class there is a total of 350 possible points. These points come from 5 homework sets that are worth 10 points each and 3 hour exams that are worth 100 points each. A student has received homework scores of 4, 8, 7, 7, and 9 and the first two exam scores are 78 and 83. Assuming that grades are assigned according to the standard scale and there are no weights assigned to any of the grades is it possible for the student to receive an A in the class and if so what is the minimum score on the third exam that will give an A? What about a B? Using the standard scale if the grade percentage is 0.9 or higher the student will get an A. Likewise, if the grade percentage is between 0.8 and 0.9 the student will get a B. (8 marks)

## Solutions to Exam

1(a)i. If  $U$  is the universal set and  $A$  is a subset of  $U$ , then all elements of  $U$  which do not belong to  $A$  form the complement of  $A$ . The complement of  $A$  is denoted by  $A^c$ .

1(a)ii. Let  $A$  and  $B$  be two sets. Then the set of elements which are common to both  $A$  and  $B$  is called the intersection of  $A$  and  $B$ , denoted by  $A \cap B$ .

1(a)iii. The union of two sets  $A$  and  $B$  is the set of all elements contained in  $A$  or  $B$ . The union of two sets  $A$  and  $B$  is denoted by  $A \cup B$ .

1b.

$$\begin{aligned} a &= 1 \\ d &= 1 \\ n &= 6 \\ S_n &= \frac{n}{2}(2a + (n-1)d) \\ &= \frac{6}{2}(2(1) + (6-1)1) \\ &= 3(2+5) \\ &= 21 \end{aligned}$$

1(c)i.  $x = 4$  or  $x = -3$

1(c)ii.  $x = -4$  and  $x = -10$

1d.

$$\begin{aligned} &= 3^2 y^4 t^2 \div \frac{1}{(3yt^2)^2} \\ &= 9y^4 t^2 \div \frac{1}{(3^2 y^2 t^4)} \\ &= (9y^4 t^2)(9^2 y^2 t^4) \\ &= 81y^6 t^6 \end{aligned}$$

1(e)i.

$$y' = 1500x^{99} - 36x^{11} + 5$$

(3 marks)

1(e)ii.

$$\begin{aligned} h(t) &= 2t^3 + 1 - 5t^{-2} \\ h'(t) &= 6t^2 + 10t^{-3} \end{aligned}$$

(4 marks)

2(a)i.

$$\begin{aligned} \text{Predicted profit} &= 120000(1.05)^{4-1} \\ &= 138915 \end{aligned}$$

2(a)ii.

$$\begin{aligned}
 u_n &> 200000 \\
 120000(1.05)^{n-1} &> 200000 \\
 1.05^{n-1} &> \frac{5}{3} \\
 \log(1.05)^{n-1} &> \log\left(\frac{5}{3}\right) \\
 (n-1)\log(1.05) &> \log\left(\frac{5}{3}\right) \\
 n &> \frac{\log\frac{5}{3}}{\log 1.05} + 1 \\
 n &> 11.6 \\
 \text{least } n &= 12 \\
 \text{year} &= 2013 + 11 = 2014
 \end{aligned}$$

2(a)iii. There are 11 terms in the sequence

$$\begin{aligned}
 \text{Total profit} &= S_n = \frac{a(r^n - 1)}{r - 1} \\
 S_{11} &= \frac{120000(1.05^{11} - 1)}{1.05 - 1} \\
 &= \$1704814
 \end{aligned}$$

2(b)i. This is an AP with a common difference of 20.

$$\begin{aligned}
 a &= 200 \\
 d &= 20 \\
 n^{\text{th}} \text{ term} &= a + d(n - 1) \\
 600 &= 200 + 20(N - 1) \\
 400 &= 20N - 20 \\
 N &= 21
 \end{aligned}$$

2(b)ii. There are 31 weeks

$$\begin{aligned}
 S_n &= \frac{n}{2}(2a + (n - 1)d) \\
 \text{or } S_n &= \frac{n}{2}(a + l) \\
 \text{Total} &= S_{21} + 600 * 32 \\
 &= \frac{21}{2}(200 + 600) + 600 * 32 \\
 &= 27000
 \end{aligned}$$

3a. There are three possible ways;

5 men and 0 women

4 men and 1 woman

3 men and 2 women

Hence;

$$\begin{aligned} & \binom{7}{5} * \binom{6}{0} + \binom{7}{4} * \binom{6}{1} + \binom{7}{3} * \binom{6}{2} \\ &= 21 * 1 + 35 * 36 + 35 * 15 \\ &= 21 + 210 + 525 = 756 \end{aligned}$$

**3b.** plugging in  $x = 2$  gives  $\frac{0}{0}$  therefore we simplify the function by factoring both the numerator and the denominator.

$$\begin{aligned} &= \lim_{x \rightarrow 2} \frac{(x-2)(x+6)}{x(x-2)} \\ &= \lim_{x \rightarrow 2} \frac{x+6}{x} \\ &= \frac{8}{2} \\ &= 4 \end{aligned}$$

**3c.**

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ 3 & -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 12 \end{bmatrix}$$

$$AX = B$$

Determinant of A = 10

Matrix of minors is given as;

$$\begin{bmatrix} 1 & 7 & -5 \\ -3 & -1 & 5 \\ 1 & -3 & 5 \end{bmatrix}$$

The matrix of co-factors;

$$\begin{bmatrix} 1 & -7 & -5 \\ 3 & -1 & -5 \\ 1 & 3 & 5 \end{bmatrix}$$

The transpose is;

$$\begin{bmatrix} 1 & 3 & 1 \\ -7 & -1 & 3 \\ -5 & -5 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{10} \begin{bmatrix} 1 & 3 & 1 \\ -7 & -1 & 3 \\ -5 & -5 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \\ 12 \end{bmatrix} = \frac{1}{10} \begin{bmatrix} 30 \\ 10 \\ 20 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

Therefore

$$x_1 = 3 \quad x_2 = 1 \quad x_3 = 2$$

4(a)i. We introduce the set  $U$  of all surveyed people and its subsets  $N$ ,  $T$ , and  $F$  of all people who read Newsweek, Time, and Fortune, respectively. The conditions of the problem then can be written as;

$$n(U) = 60$$

$$n(N) = 25, \quad n(T) = 26, \quad n(F) = 25,$$

$$n(N \cap F) = 9, \quad n(N \cap T) = 11, \quad n(F \cap T) = 8$$

$$n(N' \cap T' \cap F') = n[(N \cup T \cup F)'] = 8$$

$$n(N \cup T \cup F) = n(N) + n(T) + n(F) - n(N \cap T) - n(N \cap F) - n(T \cap F) + n(N \cap T \cap F)$$

$$n(N \cup T \cup F) = n(U) - n[(N \cup T \cup F)'] = 60 - 8 = 52$$

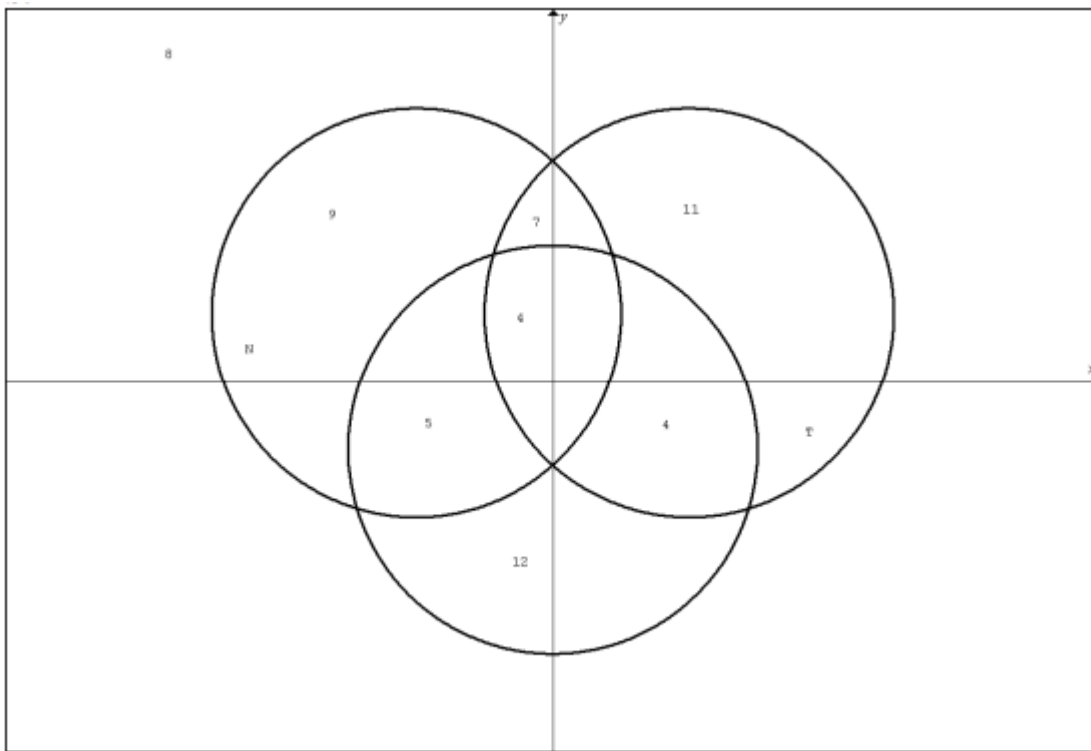
Plugging in the numbers in the formula, we have;

$$52 = 25 + 26 + 25 - 11 - 9 - 8 + n(N \cap T \cap F),$$

$$\text{hence } n(N \cap T \cap F) = 4$$

Four people read all the magazines.

4(a)ii.



(6 marks)

Figure 1: Venn diagram

4(a)iii. From the Venn diagram we see that the number of people reading exactly one of these two magazines is

$$9 + 11 + 12 = 32$$

4b. Since  $(x + y)(x + y) = x^2 - y^2$  then

$$\begin{aligned} &= (\sqrt{20})^2 - (\sqrt{5})^2 \\ &= 20 - 5 \\ &= 15 \end{aligned}$$

4c.

$$\begin{aligned} &= \frac{5^2 a^2}{b^{2 \cdot 2}} \\ &= \frac{25 a^2}{b^4} \end{aligned}$$

5a. The revenue and profit functions are given as;

$$\begin{aligned} R(x) &= x(200 - 0.005x) = 200x - 0.005x^2 \\ P(x) &= 200x - 0.005x^2 - (75000 + 100x - 0.03x^2 + 0.000004x^3) \\ &= -75000 + 100x + 0.025x^2 - 0.000004x^3 \end{aligned}$$

The marginal functions are given as;

$$\begin{aligned} C'(x) &= 100 - 0.06x + 0.000012x^2 \\ R'(x) &= 200 - 0.01x \\ P'(x) &= 100 + 0.05x - 0.000012x^2 \end{aligned}$$

The marginal costs when 2500 widgets are sold are;

$$C'(2500) = 25, \quad R'(2500) = 175, \quad P'(2500) = 150$$

The marginal costs when 7500 widgets are sold are;

$$C'(7500) = 325, \quad R'(7500) = 125, \quad P'(7500) = -200$$

5b. Let  $p$  be the minimum required score on the third exam.

Since there are no weights on the grades, the grade will be computed using the formulae;

$$\text{grade percentage} = \frac{\text{actual points}}{\text{total points}} \quad (1)$$

And we know the total points is 350 and the student has a total points including the third exam of;

$$4 + 8 + 7 + 7 + 9 + 78 + 83 + p = 196 + p$$

The smallest possible percentage for an A is 0.9 and so if  $p$  is the minimum required score on the third exam for an A we will have the following equation.

$$0.9 = \frac{196 + p}{350}$$



This is a linear equation that we will need to solve for  $p$ .

$$196 + p = 0.9(350) = 315$$

$$\implies p = 315 - 196 = 119$$

So, the minimum required score on the third exam is 119. This is a problem since the exam is worth only 100 points. In other words, the student will not be getting an A in the Algebra class. Now let's check if the student will get a B. In this case the minimum percentage is 0.8. So, to find the minimum required score on the third exam for a B we will need to solve,

$$0.8 = \frac{196 + p}{350}$$

Solving for  $p$  gives;

$$196 + p = 0.8(350) = 280$$

$$\implies p = 280 - 196 = 84$$

So, it is possible for the student to get a B in the class. All that the student will need to do is get at least an 84 on the third exam.