The Co-operative University of Kenya

SUPPLEMENTARY EXAMINATIONS

EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN STATISTICS AND INFORMATION TECHNOLOGY

COURSE CODE: BSTA 2206

UNIT TITLE: PROBABILITY AND STATISTICS III

DATE: AUGUST, 2022 Time: 2 Hours

INSTRUCTIONS

1. Answer question **ONE** and any other **TWO (2)** questions
3. Scientific Calculators and non-programmable calculators may be used
4. Use the provided statistical tables where applicable

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1. (a) State any two properties that a bivariate joint probability density function of continuous random variables X and Y should satisfy (**2** *marks*)

   (b) Show that $Cov(XY) = E(XY) - E(X)E(Y)$ (**5** *marks*)

   (c) Suppose X and Y are discrete random variables with joint probability distribution function

   $$f(x,y) = \begin{cases} k(x+2y), & x = 1,2,3; y = 0,1,3 \\ 0, & \text{elsewhere} \end{cases}$$

   Find the constant k (**5** *marks*)

   (d) Let X and Y be jointly (bivariate) noral with $Var(X) = Var(Y)$. Show that the two random variables $X+Y$ and $X-Y$ are independent. (**4** *marks*)

   (e) Give the function $f(x,y) = p^{x+y}(1-p)^{2-x-y}$. Find

      i. $E(X)$ (**4** *marks*)

      ii. $E(X+Y)$ (**4** *marks*)

      iii. $E(XY)$ (**2** *marks*)

   (f) Differentiate between CDF and PMF giving an example in each case (**4** *marks*)

2. (a) Let X and Y be two jointly continuous random variables with joint PDF

   $$f_{XY}(x,y) = \begin{cases} 2, & y+x \leq 1, x>0, y>0 \\ 0, & \text{elsewhere} \end{cases}$$

   Find $Cov(X,Y)$ and $\rho(X,Y)$ (**10** *marks*)

   (b) Give the probability density function as $f(x,y) = \begin{cases} 4xy, & 0<x<1, 0<y<1 \\ 0, & \text{otherwise} \end{cases}$

   Compute:

i. $E(X)$ **(4 marks)**

ii. $E(XY)$ **(2 marks)**

iii. $E(3X+2Y)$ **(4 marks)**

3. (a) Suppose that a radioactive particle is randomly located in a square with sides of unit length. That is, if two regions of equal areas are considered, the particle is equally likely to be in either. Let X_1 and X_2 denote the coordinates locating the particles. A reasonable model for the relative frequency histogram for $X-1$ and X_2 would be the bivariate analogues to the univariate uniform distribution.

$$f(x_1,x_2)=\begin{cases}1, & ,0\le x_1\le 1, 0\le x_2\le 1\\ 0, & \text{elsewhere}\end{cases}$$

i. Sketch the probability density surface **(2 marks)**

ii. Find $F(0.2,0.4)$ **(5 marks)**

iii. Find $P(0.1\le x_1\le 0.3; 0\le x_2\le 0.5)$ **(3 marks)**

(b) Consider the distribution of X_1 and X_2 given below

	0	1	2	$P_2(x_2)$
0	0	3/15	3/15	6/15
1	2/15	6/15	0	8/15
2	1/15	0	0	1/15
$P_1(x_1)$	3/15	9/15	3/15	1

Find the conditional distribution of X_1 given that $X_2=1$ **(7 marks)**

(c) Random variables X_1 and X_2 have the joint probability density function

$$f(x_1,x_2)=\begin{cases}4x_1x_2, & 0\le x_1\le 1; 0\le x_2\le 1\\ 0, & elsewhere\end{cases}$$

Show that X_1 and X_2 are independent **(3 marks)**

4. (a) Let X and Y be two independent discrete random variables with the same CDFs F_X and F_Y. Define

$$Z=max(X,Y)$$
$$Z=min(X,Y).$$

Find the CDFs of Z and W. **(10 marks)**

(b) Let X, Y $\sim$ Geometric (p) be independent, and let $Z=\frac{X}{Y}$.

i. Find the range of Z **(2 marks)**

ii. Find the PMF of Z **(4 marks)**

iii. Find $E(Z)$ **(4 marks)**