The Co-operative University of Kenya

END OF SEMESTER EXAMINATIONS

EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN STATISTICS AND INFORMATION TECHNOLOGY

COURSE CODE: BSTA 3221

UNIT TITLE: THEORY OF ESTIMATION

DATE: APRIL, 2023 Time: 2 Hours

INSTRUCTIONS

1. Answer question **ONE** and any other **TWO (2)** questions
3. Scientific Calculators and non-programmable calculators may be used
4. Use the provided statistical tables where applicable

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1. (a) Explain the following properties of point estimators

   i. Unbiased (**2** *marks*)

   ii. Consistent (**2** *marks*)

   iii. Efficient (**2** *marks*)

   iv. Sufficient (**2** *marks*)

   (b) Let X_1, X_2 be a random sample of size 2 from a normal distribution with mean μ and variance 5. Consider the following three estimators of μ:
   $t_1 = \frac{2}{3}X_1 + \frac{1}{3}X_2$, $t_2 = \frac{1}{2}X_1 + \frac{3}{4}X_2$, $t_3 = \frac{3}{2}X_1 - \frac{1}{2}X_2$
   Which one of the three estimators t_1, t_2 and t_3 would you recommend for use as an estimator of μ. Give reasons. (**7** *marks*)

   (c) Explain Cramer-Rao Inequality (CRI) (**2** *marks*)

   (d) A random sample $X_1, X_2, ..., X_n$ of size n is taken from a normal population with mean μ and variance σ^2 both unknown. For a random sample of size 25, it was found that $\sum_{i=1}^{25} X_i = 3700$ and $\sum_{i=1}^{25} X_i^2 = 573,000$.

   a. Obtain point estimates for μ and σ^2 (**3** *marks*)

   b. Obtain a 98% confidence intervals for μ and σ^2 (**5** *marks*)

   c. Calculate a new confidence interval for μ assuming σ^2 was known to be 800 (**5** *marks*)

2. (a) Given the following data, use matrix notation to fit a

   i. straight line $y = \alpha + \beta x$ (**4** *marks*)

   ii. quadratic equation $y = \beta_0 + \beta_1 x + \beta_2 x^2$ (**6** *marks*)

X	-2	-1	0	1	2
Y	0	0	1	1	3

   (b) Define the method of maximum likelihood (**3** *marks*)

(c) Let $X_1, X_2, ..., X_3$ be a random sample of size n from a Poisson distribution with parameter λ. Find the MLE of λ (**7** *marks*)

3. (a) Let $X_1, X_2, ..., X_n$ be a random sample of size n from a normal population with mean μ and variance σ^2. Find the minimum variance unbiased estimators of

   i. μ when σ^2 is known (**3** *marks*)

   ii. σ^2 when μ is known (**3** *marks*)

   (b) A manufacturer of gunpowder has developed a new powder that was tested in eight shells. The resulting muzzle velocities in meters per second were as follows:

   3005, 2925, 2935, 2965, 2995, 3005, 2937, 2905

   Find the confidence interval for the true average, μ, for the shells of this type with a confidence coefficient of 0.95. Assume muzzle velocities are approximately normally distributed. (**6** *marks*)

   (c) Let $X_1, X_2, ..., X_n$ be a random sample of size n from a Poisson distribution. Show that $T = \sum_{i=1}^{n}$ is a sufficient statistic for λ (**8** *marks*)

4. (a) Differentiate between Loss and Risk functions (**4** *marks*)

   (b) Let $X_1, X_2, ..., X_n$ be a random sample of size n from a population whose density function with parameter $\theta > 0$ is given by $f(X) = \begin{cases} \theta^x (1-\theta) & , x = 0, 1 \\ 0 & , \text{otherwise} \end{cases}$

   If an estimator of θ is given by $\hat{\theta} = \frac{1}{n}\sum_{i=1}^{n} X_i$, test whether $\hat{\theta}$ is an

   i. Unbiased (**3** *marks*)

   ii. Consistent estimator of θ (**4** *marks*)

   (c) Let $X_1, X_2, ..., X_n$ be a random sample of size n from a normal population with mean μ and variance σ. Find the moment estimators of μ and σ^2 (**9** *marks*)

5. (a) Let $X_1, X_2, ..., X_n$ be a random sample of size n from a Poisson distribution with parameter λ. Find MVUE of λ and verify that it is indeed UMVUE (**5** *marks*)

   (b) The characteristic equation for a statistic $T = t(X_1, X_2, ..., X_n)$ to be UMVUE of $\omega(\theta)$ is $T = \omega(\theta) + \frac{\omega(\theta)}{I(\theta)} \frac{dlnL}{d\theta}$. Proof (**15** *marks*)

t Table

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	**0.50**	**0.25**	**0.20**	**0.15**	**0.10**	**0.05**	**0.025**	**0.01**	**0.005**	**0.001**	**0.0005**
two-tails	**1.00**	**0.50**	**0.40**	**0.30**	**0.20**	**0.10**	**0.05**	**0.02**	**0.01**	**0.002**	**0.001**
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
	Confidence Level										

t-table.xls 7/14/2007