The Co-operative University of Kenya

Special EXAMINATIONS

EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN STATISTICS

COURSE CODE: BSTA 3221

UNIT TITLE: THEORY OF ESTIMATION

DATE: July, 2022 Time: 2 Hours

INSTRUCTIONS

1. Answer question **ONE** and any other **TWO (2)** questions
3. Scientific Calculators and non-programmable calculators may be used
4. Use the provided statistical tables where applicable

QUESTION ONE (THIRTY MARKS)

1. Explain the following terms as used in statistical estimation; (8marks)

   (a) A parameter

   (b) A statistic

   (c) An estimator

   (d) An estimate

2. State Neyman-Fisher factorization criterion for testing sufficiency of a statistic as used in statistical estimation. (3 marks)

3. Let $x_1, x_2, x_3 ... x_n$ be independently and identically distributed random variables with poisson distribution probability density function;

Show that the sample mean is a sufficient estimator for the parameter λ (5 marks)

4. Suppose that there's a random sample $x_1, x_2, x_3, ...x_n$ of size n from a population with mean μ

   , μ show that the sample mean $\bar{X}$ is unbiased estimatorof μ

   4 marks)

5. Manufacturer of gun has developed a new powder which was tested in eight shells. The resulting muzzle velocities in metres per second were as follows: 3105

   3925 3935 2965 2895 3205 2937 2905

   Find the confidence interval for the true average, μ , for the shells of this type with confidence coefficient of 0.95. assume muzzle velocities are approximately normally distributed. (6 marks)

6. Distinguish between the point estimation and interval estimation (4 marks)

QUESTION TWO (TWENTY MARKS)

1. Distinguish between the terms sufficiency and consistency as used in theory of estimation. (4 marks)

2. A random sample of 5 observations $x_1 x_2, ..., x_5$ is taken from a normal population with mean μ and variance σ^2 both unknown. Consider the following estimators of μ :

   (a)
   $$t_1 = \frac{1}{10}\sum_{i=1}^{5} x_i^3$$

   (b)
   $$t_2 = \frac{x_3 + x_2}{2} + x_1$$

   (c)
   $$t_3 = \frac{2x_1 + x_3 + \lambda x_3}{3}$$
   where λ is a constant and t_3 is unbiased estimator.

Find λ , state giving reasons the estimator which is best estimator among $t_1, t2, t_3$. (10 marks)

3. Obtain the Maximum likelihood estimator (MLE) for the parameter λ in a Poisson population given by

$$f(x) = \begin{cases} \dfrac{e^{-\lambda}\lambda^x}{x!}, x = 0, 1, 2, 3, \ldots \\ 0, elsewhere \end{cases}$$ (6 marks)

QUESTION THREE (TWENTY MARKS)

1. Students in STA 3115 class claimed that doing there assignment had not helped them prepare for the main exam. The exam score Y and assignment score X for 4 students were as follows

X	Y	XY	X^2
296	195		
301	280		
278	179		
264	169		

Complete the table and hence obtain β_0 and β_1 and write prediction equation (12marks)

4. Obtain the method of moments estimator for λ in a Poisson distribution given by the density function (8 marks)

$$f(x) = \begin{cases} \dfrac{e^{-\lambda}\lambda^{x}}{x!}, x = 0, 1, 2, 3, \ldots \\ 0, elsewhere \end{cases}$$

QUESTION FOUR (TWENTY MARKS)

1. $x_1, x_2, x_3, \ldots\ldots\ldots\ldots\ldots x_n$ is random sample from a population given by

$$f(x) = \begin{cases} 1, \lambda - \dfrac{1}{2} < x < \lambda + \dfrac{1}{2} \\ 0, elsewhere \end{cases}$$

Prove that $\bar{x}$ is a consistent estimator for λ (10 marks)

2. Obtain the MVUE for μ in the normal population with probability density

$$f(x) = \begin{cases} \dfrac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}, -\infty < x < \infty, -\infty < \mu < \infty \\ 0, elsewhere \end{cases}$$

where σ^2 is known. (10 marks)

QUESTION FIVE (TWENTY MARKS)

5. Let $x_1, x_2, x_3, ..., x_n$ be a random sample of size n from a population whose probability density function is;

$$f(x, \theta) = \begin{cases} \theta e^{-\theta x}, x > 0 \\ 0, otherwise \end{cases}$$

Taking $\Phi(\theta) = (\theta)$, find the Cramer Rao lower bound for (θ). (10 marks)

6. If 8.0, 7.5, 8.7, 7.4, 8.4, 9.8, 6.2, 7.8, 8.5 are observed values of a random variable which is distributed normally with mean 8 and unknown variance σ^2.Construct 98% confidence intervals for σ^2. (10 marks)